

# Generation Ramping Valuation in Day-Ahead Electricity Markets

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**Abstract**—In this paper, we first introduce a variational formulation of the Unit Commitment (UC) problem, in which generation and ramping trajectories of the generating units are continuous time signals and the generating units cost depends on the three signals: the binary commitment status of the units as well as their continuous-time generation and ramping trajectories. We assume such bids are piecewise strictly convex time-varying linear functions of these three variables. Based on this problem derive a tractable approximation by constraining the commitment trajectories to switch in a discrete and finite set of points and representing the trajectories in the function space of piece-wise polynomial functions within the intervals, whose discrete coefficients are then the UC problem decision variables. Our judicious choice of the signal space allows us to represent cost and constraints as linear functions of such coefficients; thus, our UC models preserves the MILP formulation of the UC problem. Numerical simulation over real load data from the California ISO demonstrate that the proposed UC model reduces the total day-ahead and real-time operation cost, and the number of ramping scarcity events in the real-time operations.

**Index Terms**—Unit commitment, generation trajectory, ramping trajectory, ramping cost, continuous-time function space, mixed-integer linear programming.

## I. INTRODUCTION

THE important task of a functional electricity market is not only to schedule generating units to supply the load considering their inherent capacity limitations, but also taking into account their limits in ramping up and down and follow the vagaries of the load profile. The market clearing is handled by solving the unit commitment (UC) problem which schedules the most economic set of generating units on an hourly basis, to meet the hourly forecasted load of the next day. The current UC practice has worked well for compensating the variability and uncertainty of load in the past. However, this practice is starting to fall short, as increasing renewable energy resources add variability to the system and large ramping events occur much more frequently [1]–[4].

Ramping events and constraints are inter-temporal continuous-time mathematical objects. The natural implication of the current hourly UC practice is that, within the hour, generating units shall follow a linear ramp from one value to the next. Intuitively, the linear ramping does not fully capture the prior information about sub-hourly variations of

the net-load and one must expect deviation which will have to be handled in the real-time operation [5]. If this short-term deviation is beyond the coverage of the hourly day-ahead dispatch decisions, the short-term operations may be left with sufficient capacity but without ramping capability to respond to sub-hourly net-load variations, as referred to ramping scarcity events by multiple ISOs [6], [7], with obviously undesirable economic and security consequences.

With the increasing critical demand for ramping capacity, many believe that generating units must be remunerated explicitly not only for their capacity but also for their ramping capability, in order to stimulate competition and compensate the additional wear and tear cost that generating units incur due to more frequent ramping [8], [9]. However, there is disagreement on how to do so in the most effective fashion. There have been notable research efforts on developing new operation models, market mechanisms and services to better taking care of renewable generation ramping and compensating the resources for providing the additional ramp. For example, in [10], a security-constrained UC algorithm was developed that takes into account the intermittency and volatility of wind power generation. A day-ahead UC model with stochastic security was formulated in [11] which is capable of accounting for non-dispatchable and variable wind power generation. A methodology is proposed in [12] to determine the required level of various reserves in a power system with a high penetration of wind power. A formulation is developed in [13] that models the dynamic ramping and startup costs of generating units in the UC problem. An attempt is made in [14] to model the power trajectory of thermal generating units as a piecewise linear function, and calculate the ramping cost as a proportion of the energy produced in the ramping process.

Most recently, the Midcontinent ISO (MISO) and the California ISO (CAISO) have proposed new flexible ramping products to address this operational challenge. In the MISO, the flexible ramping product is designed to cover the net-load uncertainty in the next 10 minutes [6], [15]. In the CAISO, the flexible ramping product is designed to provide load following flexibility for the next 5 minutes and may look ahead several intervals [7], [16]. In [17], an optimization-based model is used to evaluate the ramping capability requirement considering both security and economics. In [18], a deterministic ramping capability model with transmission constraint is proposed to ensure its deliverability. In [19], both the deterministic and stochastic models are evaluated in designing the market for flexible ramping products. In [20], a robust economic dispatch model is developed with ramping capability requirement and compared with the deterministic model. A stochastic day-

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ahead scheduling model is developed in [21], where the flexible ramping product is scheduled in the day-ahead for managing the variability of renewable generation. However, defining new ramping services, like the flexible ramping product, without incorporating additional information about sub-hourly ramping of the net-load complicates the market structure, and raises questions about what is the reasonable level of cost allocation on these new market products.

Our hypothesis in this work is that a crucial bottleneck lies in the prevalent representation of the scheduling decision space in the UC problem, and the structure of the generating units' cost functions in the day-ahead operation. In the current UC practice, the generating units only bid and get compensated for energy generation. The UC decision space includes *only* hourly commitment decision points and hourly generation schedules, which form a piecewise constant generation trajectory for each generating unit. These piecewise constant trajectories are a zero-order approximation of the higher-order continuous-time trajectories that correspond to the actual UC problem decision space. In addition, a continuous-time trajectory has potentially infinite derivatives that could be chosen as part of the optimization and, if the derivatives was taken into consideration and appropriately priced, it could alter the priority given to different units in the schedule.

In fact, the real-time load deviation results from two kinds of error in day-ahead load profile: 1) error due to the imperfect forecast, 2) error due to the day-ahead load profile approximation. The main goal of our proposed model is to reduce the second kind of error, capturing more accurately the information available about the net-load evolution in time in the day-ahead operation, while revealing in the commitment stage the potential operational flexibility of generating units that can have significant impact on the day-ahead operation solution. A more flexible generation trajectory model taps on additional sub-hourly ramping flexibility of generating units that is not captured by current UC formulation. In order to address the increased ramping demand, instead of limiting the decision space to the commitment state and generation trajectory, it would be advantageous to also include the first derivative of the generation trajectory, i.e. the ramping trajectory, as the decision variable and among the degrees of freedom, opening the door to receiving competitive offers that capture the joint cost of generation and of generation ramping at each point.

Our idea, in a nutshell, is to change the UC formulation in two ways: 1) expanding the decision space by modeling generation trajectories with piecewise polynomial functions of degree 3, and defining the corresponding ramping trajectories with piecewise polynomial functions of degree 2, which offer sufficient flexibility to match the sub-hourly variations of demand; 2) generalizing the generating units operation cost function as continuous-time convex function of both generation value and the ramp. While considering higher order polynomial expansions would be a simple generalization of our idea, using polynomials of degree three allows us to constrain the decision space to have not only continuous generation but also continuous ramping trajectories, while harnessing additional degrees of freedom that the current UC problem ignores of choosing.

Our numerical results in Section VI are promising and suggest that we can, indeed, reduce real time and total operation costs, supporting the intuition that there is an economic benefit in curbing decision errors that are entirely *predictable* and that are simply due to a poor approximation of the net-load forecast. But the numerical simulations are limited and we will need further analysis to determine if we perform uniformly better, considering the ramping products and other operation stages in the cost to address the sub-hourly variations.

The rest of the paper is organized as follows: in Section II, we propose that the UC problem is originally a constrained variational problem, and present a function space model to convert the variational problem to a discrete-time optimization problem. We then propose in Section III to use Bernstein polynomials to approximate the continuous-time UC decision space. The generalized operation cost function of generating units is developed in Section IV. The proposed UC model is presented in Section V. The numerical simulations using the CAISO's net-load data is presented in Section VI, and finally the conclusions are discussed in Section VII.

## II. CONTINUOUS-TIME UNIT COMMITMENT PROBLEM

We consider a day-ahead electricity market setting in which a set of  $K$  generating units compete to sell continuous-time generation trajectory  $G_k(t)$  forming a vector  $\mathbf{G}(t) = (G_1(t), \dots, G_K(t))^T$  and supply the forecasted load profile  $N(t)$  over the day-ahead operating horizon  $\mathcal{T}$  at minimum cost, subject to prevailing constraints of the generating units, including capacity, ramping and minimum on/off time constraints. Along with the generation trajectories, the dispatch decisions determine also the units ramping trajectories  $\mathbf{G}'(t) = (G'_1(t), \dots, G'_K(t))^T$ . The generating units also have a commitment variable  $I_k(t)$  forming the vector  $\mathbf{I}(t) = (I_1(t), \dots, I_K(t))^T$  that is a step from 0 to 1 when the unit is started up, and from 1 to 0 when the unit is shut down. Let us also assume that the generating unit  $k$  expresses a cost function  $C_k(G_k(t), G'_k(t), I'_k(t); t)$  such that  $C_k(G_k(t), G'_k(t), I'_k(t); t)dt$  is the incremental cost the unit experiences for generating the power  $G_k(t)$  with a slope  $G'_k(t)$  at time  $t$ , plus possible costs associated with the change in state  $I'_k(t)$ , such as startup and shutdown cost. We will assume that the cost function has the following two contributions:

$$C_k(G_k(t), G'_k(t), I'_k(t); t) = C_k^I(I'_k(t); t) + C_k^{G, G'}(G_k(t), G'_k(t); t), \quad (1)$$

where the first term is:

$$C_k^I(I'_k(t); t) = \max(c_k^{(\text{SU})}(t)I'_k(t), -c_k^{(\text{SD})}(t)I'_k(t)), \quad (2)$$

and represents the startup and shutdown costs, while  $C_k^{G, G'}(G_k(t), G'_k(t))$  is a convex function with respect to the pair  $(G_k(t), G'_k(t))$ ; both terms are function of the time as well. In (2),  $c_k^{(\text{SU})}$  and  $c_k^{(\text{SD})}$  are respectively the startup and shutdown costs of generating unit  $k$ .

Ideally, the continuous-time generation, ramping and commitment variables  $G_k(t)$ ,  $G'_k(t)$ ,  $I_k(t)$  can change in any instant of time  $t \in \mathcal{T}$  giving the ultimate flexibility to the generation fleet to match the load and thus resulting in the truly

optimal solution for the UC problem. In terms of the binary commitment variable, this means that the units are flexible to turn on and off on at any time instant  $t \in \mathcal{T}$  such that the minimum on and off time constraints are met. Thus the  $I_k(t)$  can be expressed as a sum of unit step functions as follows:

$$I_k(t) = \sum_{h=1}^{H_k} \left( u(t - t_{k,h}^{(\text{SU})}) - u(t - t_{k,h}^{(\text{SD})}) \right), \quad (3)$$

where  $H_k$  is the total number of startup/shutdown cycles of the generating unit  $k$  over  $\mathcal{T}$ , and  $t_{k,h}^{(\text{SU})}$  and  $t_{k,h}^{(\text{SD})}$  respectively denote the times when the unit  $k$  starts up and shuts down in cycle  $h$ . In this representation, the times  $t_{k,h}^{(\text{SU})}$  and  $t_{k,h}^{(\text{SD})}$  are the decisions variables of the UC optimization problem. The derivative of the commitment variable becomes:

$$I'_k(t) = \sum_{h=1}^{H_k} \left( \delta(t - t_{k,h}^{(\text{SU})}) - \delta(t - t_{k,h}^{(\text{SD})}) \right), \quad (4)$$

where  $\delta(t)$  is Dirac delta function. The resulting continuous-time UC problem for the day-ahead market clearing is a constrained variational problem in nature, expressed as follows:

$$\min \sum_{k=1}^K \int_{\mathcal{T}} C_k(G_k(t), G'_k(t), I'_k(t)) dt \quad (5)$$

$$\text{s.t.} \sum_{k=1}^K G_k(t) = N(t) \quad \forall t \in \mathcal{T} \quad (6)$$

$$\underline{G}_k I_k(t) \leq G_k(t) \leq \overline{G}_k I_k(t) \quad \forall k, t \in \mathcal{T} \quad (7)$$

$$\underline{G}'_k I'_k(t) \leq G'_k(t) \leq \overline{G}'_k I'_k(t) \quad \forall k, t \in \mathcal{T} \quad (8)$$

$$t_{k,h}^{(\text{SD})} - t_{k,h}^{(\text{SU})} \geq T_k^{(\text{on})} \quad \forall k, h, t \in \mathcal{T} \quad (9)$$

$$t_{k,h+1}^{(\text{SU})} - t_{k,h}^{(\text{SD})} \geq T_k^{(\text{off})} \quad \forall k, h, t \in \mathcal{T} \quad (10)$$

where  $T_k^{(\text{on})}$  and  $T_k^{(\text{off})}$  are the minimum on and off time limits of the generating unit  $k$ . The objective in (5) is to minimize the total continuous-time cost of generating units over the scheduling horizon  $\mathcal{T}$ , subject to the continuous-time load-generation balance constraint (6), the continuous-time generation capacity and ramping constraints in (7) and (8), and the minimum on/off time constraints in (9) and (10).

The ideal infinite flexibility of units to change generation, ramping and commitment status assumed in (5) would result in an informationally complex and computationally intractable variational problem (1)-(10). To overcome this impasse, we need to covert the variational problem to a discrete optimization problem, where the continuous-time trajectories and uncountable set of commitment options are respectively approximated with finite countable continuous variables and countable binary variables over the scheduling horizon, forming a discrete-time mixed-integer optimization problem.

#### A. Load Approximation

Given past observations of the net-load during previous periods in which the load trajectories are observed with high resolution during comparable seasons and time-horizons, it is reasonable to build a statistical model considering the realizations as samples of a stochastic process. Let the roman letter  $N(t; \omega)$  denote a realization of such process, where  $\omega$

is the index of the particular outcome in the sample space of load trajectories  $\Omega$ . Assume that during  $\mathcal{T}$ , except for a small residual error, all forecasts lie on a countable and finite signal space of dimensionality  $P$ , spanned by a set of bases functions  $\mathbf{e}(t) = (e_1(t), \dots, e_P(t))^T$ , that is:

$$N(t) = \mathbf{N}^T \mathbf{e}(t) + \epsilon_N(t) \quad (11)$$

where we indicate with  $\mathbf{N} = (N_1, \dots, N_P)^T$  the coordinates of the approximation onto the subspace spanned by  $\mathbf{e}(t)$ . The approximation with the smallest Frobenius norm for the error would be to choose  $\mathbf{N}^T \mathbf{e}(t)$  as the orthogonal projection of  $N(t)$  onto the space spanned by  $\mathbf{e}(t)$ . By *small* residual error we mean that the residual uncertainty on the load is the dominant source of incongruence between the day-ahead UC schedule and the real-time operation decision, however we assume in this paper that there exist an algorithm that provides this mapping and leave further analysis to future work<sup>1</sup>.

The highlight here is that we can decompose any generation trajectory also into a component that lies in the subspace spanned by  $\mathbf{e}(t)$  and in a component orthogonal to it, i.e.:

$$G_k(t) = \mathbf{G}_k \mathbf{e}(t) + \epsilon_{G_k}(t). \quad (12)$$

Considering this fact, the continuous-time load generation balance constraint in (6) is equivalent to:

$$\mathbf{N} = \sum_{k=1}^K \mathbf{G}_k, \quad \sum_{k=1}^K \epsilon_{G_k}(t) = \epsilon_N(t). \quad (13)$$

Because the approximation error is buried in the uncertainty about the net-load, the solution of the variational problem can be further approximated by assuming  $\epsilon_N(t) = 0$ , which also implies that, without further loss of optimality, we can pick the generation trajectories in the subspace spanned by  $\mathbf{e}(t)$ :

$$G_k(t) = \mathbf{G}_k \mathbf{e}(t). \quad (14)$$

It becomes customary to define the *continuous-time ramping trajectory* of generating unit  $k$  as:

$$G'_k(t) = \mathbf{G}_k \mathbf{e}'(t). \quad (15)$$

The problem here is the choice of the bases functions  $\mathbf{e}(t)$  that not only represents a good approximation of  $N(t)$ , but also is helpful in: 1) the enforcement of continuous-time generation capacity and ramping constraints, 2) the commitment of the units through the integer variables, and 3) the definition of ramping trajectory and the associated cost. In the next Section, we propose to use the cubic splines to discretize the decisions on the continuous-time generation trajectory.

<sup>1</sup>Let the roman letter  $N(t; \omega)$  denote a realization of the random net-load process, where  $\omega$  is the index of the particular outcome in the sample space of load trajectories  $\Omega$ . Let  $MSE_N = \int_{\mathcal{T}} \mathbb{E}\{\|N(t; \omega) - N(t)\|^2\} dt$ . A reasonable heuristic would be requiring that:  $\sup_{N(t)} \int_{\mathcal{T}} |\epsilon_N(t)|^2 dt \ll MSE_N \ll \inf \int_{\mathcal{T}} |N(t)|^2 dt$ , which means that the function space must include approximations for  $N(t)$  that entail an error signal whose Frobenius norm is always much smaller than the forecast mean-squared error  $MSE_N$ , which is small itself relative to the forecast. Since  $MSE_N$  is bounded by the conditional variance of  $N(t; \omega)$  given all the history we use to forecast it, in general, with reasonable regularity conditions on the functions  $N(t)$ , bases  $\mathbf{e}(t)$  can be found that, as  $P$  grows, leave a vanishing residual error.

### III. DISCRETE-TIME UC DECISION SPACE

In the current electricity market practice, the commitment variables  $\mathbf{I}(t)$  are limited to hourly changes, reducing the number of switching options to discrete (hourly) times  $t_0, \dots, t_I$ . Dividing the horizon  $\mathcal{T}$  in  $I$  intervals  $\mathcal{T}_i = [t_i, t_{i+1})$ ,  $\rightarrow \mathcal{T} = \cup_{i=0}^{I-1} \mathcal{T}_i$ , the continuous-time commitment variable is:

$$I_k(t) = \sum_{i=0}^{I-1} I_{k,i} [u(t - t_i) - u(t - t_{i+1})] \quad (16)$$

$$= \sum_{i=0}^{I-1} (I_{k,i} - I_{k,i-1}) u(t - t_i) \quad (17)$$

and is linear with respect to  $I_k(t_i) \equiv I_{k,i}$  which are the binary decision variables of the UC problem, and  $I_{k,-1}$  is given for all generating units. Even though the switching variables are constrained to remain constant during the hourly intervals, the continuous-time generation and ramping trajectories should be flexible to change between two consecutive hourly schedules.

The hourly change of generating units commitment status suggests the idea of using splines functions as a basis in each sub-interval  $\mathcal{T}_i$  to represent the whole scheduling horizon. There are several different family of splines that can be used to approximate the continuous-time trajectory (space) of a data set with the desired level of accuracy, as the order of the basis grows. Among the polynomial splines, the Bernstein polynomials of degree  $Q$  are defined as:

$$b_{q,Q}(t) = \binom{Q}{q} t^q (1-t)^{Q-q} \Pi(t), \quad t \in \mathcal{T}_i \quad (18)$$

where  $\Pi(t) = u(t-1) - u(t)$  is the rectangular pulse equal to 1 in  $[0, 1)$  and zero else, and  $q = 0, \dots, Q$ . Note that the space spanned by Bernstein polynomials of degree  $Q'$  is a subspace of the space spanned by Bernstein polynomials of any degree  $Q > Q'$  and, therefore, there exist a  $(Q+1) \times (Q'+1)$  linear mapping between the coefficients of a signal in the basis of order  $Q'$  and those in the basis of order  $Q$ . In particular, for  $Q' = 0$  when the only basis vector is  $b_{0,0}(t) = \Pi(t)$  the  $(Q+1) \times 1$  linear map is the all one vector since:

$$\sum_{q=0}^Q b_{q,Q}(t) = \sum_{q=0}^Q \binom{Q}{q} t^q (1-t)^{Q-q} \Pi(t) = \Pi(t). \quad (19)$$

When we are interested in the piecewise approximation of a set of data points, an important feature of the Bernstein polynomials is that they can be utilized to more easily impose smoothness conditions not only at the break points but also inside the interval of interest, working only on the coefficients of the Bernstein spline expansion. In addition, the convex hull property of the Bernstein polynomials enables us to enforce the capacity and ramping constraints on the continuous-time generation trajectory by capping the corresponding coefficients.

Specifically, for each of the intervals  $\mathcal{T}_i$  that are used to control the switching point, we construct a subset of basis function using the Bernstein Polynomials of order  $Q$ . So, the basis functions  $\mathbf{e}^{(Q)}(t)$  spanning the whole horizon  $\mathcal{T}$  contains  $P = (Q+1)I$  functions with the components defined as follows:

$$e_{i(Q+1)+q}^{(Q)}(t) = b_{q,Q} \left( \frac{t - t_i}{t_{i+1} - t_i} \right), \quad (20)$$

for  $i = 0, \dots, I-1; q = 0, \dots, Q$ . To reduce the notation, we define  $p \equiv i(Q+1) + q$ , where  $p$  goes from 0 to  $(Q+1)I$ .

Hence, the continuous-time load profile  $N(t)$  over the the operating horizon is such that:

$$\begin{aligned} N(t) &= \mathbf{N}^T \mathbf{e}^{(Q)}(t) + \epsilon_N(t) \\ &= \sum_{p=0}^{P-1} N_p e_p^{(Q)}(t) + \epsilon_N(t) \end{aligned} \quad (21)$$

Given  $N(t)$  is continuous in  $\mathcal{T}$ , from Weierstrass approximation theorem follows that for any  $\epsilon > 0$  there exist a positive  $Q'$  such that for all  $t \in \mathcal{T}$  and for all  $Q \geq Q'$  in (20), we have

$$|N(t) - \mathbf{N}^T \mathbf{e}^{(Q)}(t)| < \epsilon \quad (22)$$

which states that the expansion on Bernstein polynomials for sufficiently large  $Q$  converges to  $N(t)$ . In the following, we assume that for a large enough  $I$  the order  $Q = 3$  is sufficient to render the approximation error negligible and, from now on, we assume that  $\epsilon_N(t) = 0$  and  $Q = 3$ .

#### A. Load and Generation piece-wise polynomial expansion using Bernstein Spines of degree 3

Here, we propose to use the cubic spline function space of Bernstein polynomials of degree 3 for approximating continuous-time load profile. Cubic splines interpolate points with minimum curvature while providing additional flexibility to fit the continuous-time load variations. Correspondingly, the continuous-time generation trajectory is also modeled through the Bernstein polynomials of degree 3.

The day-ahead load profile in the  $4I$ -dimensional function space of Bernstein polynomials of degree 3, which can be expressed as in (21) for  $Q = 3$ :

$$N(t) = \mathbf{N}^T \mathbf{e}^{(3)}(t) = \sum_{p=0}^{P-1} N_p e_p^{(3)}(t) \quad (23)$$

where  $\mathbf{N}$  is the  $4I$ -dimension vector of coefficients. The continuous-time generation trajectory of units over the day-ahead scheduling horizon can be expressed in the function space of Bernstein polynomials of degree 3 as follows:

$$G_k(t) = \mathbf{G}_k^T \mathbf{e}^{(3)}(t) = \sum_{p=0}^{P-1} G_{k,p} e_p^{(3)}(t) \quad (24)$$

where  $\mathbf{G}_k$  is the  $4I$ -dimensional Bernstein coefficients vector of the generation trajectory of unit  $k$ .

1) *Definition of Ramping Trajectory:* One of the important properties of the Bernstein polynomials states that the derivatives of the Bernstein polynomials of degree  $Q$  can be expressed as the degree of the polynomial, multiplied by the difference of two Bernstein polynomials of degree  $Q-1$  [22]. Specifically, for degree 3 we can write:

$$b'_{q,3}(t) = 3(b_{q-1,2}(t) - b_{q,2}(t)), \quad q = 0, 1, 2. \quad (25)$$

This important property allows us to define the *continuous-time ramping trajectory* of generating unit  $k$  in a space

spanned by Bernstein polynomials of degree 2 as follows:

$$G'_k(t) = \mathbf{G}'_k{}^T \mathbf{e}^{(2)}(t) = \sum_{p=0}^{P-1} G'_{k,p} e_p^{(2)}(t) \quad (26)$$

where  $\mathbf{G}'_k$  is the vector of Bernstein coefficients of the continuous-time ramping trajectory, which can be expressed in terms of coefficients of the generation trajectory as follows:

$$\mathbf{G}'_k = \mathbf{O} \mathbf{G}_k. \quad (27)$$

where  $\mathbf{O}$  is the  $3I \times 4I$  linear matrix relating the derivatives of Bernstein polynomials of degree 3 with the Bernstein polynomials of degree 2.

2) *Continuity*: The continuity of the generation trajectory at the edge points of the intervals is guaranteed by imposing appropriate constraints on the coefficients from adjacent intervals. The  $C^0$  continuity requires the continuity of generation trajectory at the edges, i.e.  $G_{k,4i+3} = G_{k,4(i+1)}$ . The  $C^1$  continuity is assured by imposing the constraints:

$$G_{k,4i+3} = G_{k,4(i+1)}, \quad G_{k,4i+3} - G_{k,4i+2} = G_{k,4(i+1)+1} - G_{k,4(i+1)}, \quad \forall i. \quad (28)$$

Note that the  $C^1$  continuity of generation trajectory in (28) results in  $C^0$  continuity of ramping trajectory at the edge points of the intervals, i.e.,  $G'_{k,3(i+1)} = G'_{k,3i+2}$ . In essence, imposing  $C^1$  continuity implies that the generation trajectory effectively lies in a function space with dimensionality  $2I$ .

3) *Convex Hull Property*: The other useful property of Bernstein polynomials is that the continuous-time generation and ramping trajectories satisfy the *convex hull property* [22], namely that the continuous-time trajectories  $G_k(t)$  and  $G'_k(t)$  in interval  $i$  will never be outside of the convex hull of the control polygon formed respectively by the Bernstein points  $\mathbf{G}_k$  and  $\mathbf{G}'_k$ . Accordingly, the lower and upper bounds of the continuous-time generation and ramping trajectories within the interval  $i$  can be respectively represented by the associated Bernstein coefficients in (29)-(32).

$$\min_{t_i \leq t \leq t_{i+1}} \{G_k(t)\} \geq \min_{q=0,1,2,3} \{G_{k,4i+q}\}, \quad (29)$$

$$\max_{t_i \leq t \leq t_{i+1}} \{G_k(t)\} \leq \max_{q=0,1,2,3} \{G_{k,4i+q}\}, \quad (30)$$

$$\min_{t_i \leq t \leq t_{i+1}} \{G'_k(t)\} \geq \min_{q=0,1,2} \{G'_{k,3i+q}\}, \quad (31)$$

$$\max_{t_i \leq t \leq t_{i+1}} \{G'_k(t)\} \leq \max_{q=0,1,2} \{G'_{k,3i+q}\}, \quad \forall i. \quad (32)$$

4) *Commitment Variable Model using Bernstein Polynomials*: In (16), we represented the continuous-time commitment variables using two steps functions, i.e. the rectangular function, in each interval. (16) can be equivalently written as:

$$I_k(t) = \sum_{i=0}^{I-1} I_{k,i} \Pi \left( \frac{t - t_i}{t_{i+1} - t_i} \right), \quad (33)$$

where we can recognize  $\Pi \left( \frac{t - t_i}{t_{i+1} - t_i} \right)$  as being a special case of our basis construction for  $Q = 0$ , i.e.  $e_i^{(0)}(t) = b_{0,0} \left( \frac{t - t_i}{t_{i+1} - t_i} \right)$  for  $i = 0, \dots, I-1$ . The piece-wise constant continuous-time

commitment variable can be also expressed in the function space of Bernstein polynomials of degree 3 as follows:

$$I_k(t) = \mathbf{I}_k^T \mathbf{e}^{(3)}(t) = \sum_{p=0}^{P-1} I_{k,p} e_p^{(3)}(t), \quad (34)$$

where  $\mathbf{I}_k$  is the coefficients vector of the generation trajectory of unit  $k$  in the function space of Bernstein polynomial of degree 3. The four Bernstein coefficients of  $I_k(t)$  in each interval equal to 1, if the unit is committed in that period, and equal to zero if it is not committed. Thus, the two representations for  $I_k(t)$  in (33) and (34) are equivalent. The  $C^1$  continuity of generation trajectory in (28) implies that the last two coefficients in each interval are dependent to the commitment variable at the subsequent interval, while the first two coefficients are dependent to the commitment variable at the same interval. Thus, the four Bernstein coefficients in each interval  $i$  are defined as follows:

$$I_{k,4i}^{(3)} = I_{k,4i+1}^{(3)} = I_{k,i}, \quad I_{k,4i+2}^{(3)} = I_{k,4i+3}^{(3)} = I_{k,i+1}. \quad (35)$$

#### IV. GENERALIZED OPERATION COST FUNCTION

As presented in (1), the operation cost of generating units is generally a function of the generation and ramping trajectories of generating units, and includes the startup/shutdown cost term. Here, we first derive the expression for startup and shutdown cost term. We then leverage the continuous-time generation and ramping trajectory models in (24) and (26) to define the joint generation and ramping cost function.

##### A. Startup/Shutdown Cost

The cost term  $C_k^I(I'_k(t); t)$  in (2), equals to startup cost when there is a positive change in commitment variable, i.e.  $I'_k(t) \geq 0$ , and it equals to shutdown cost when  $I'_k(t) \leq 0$ . Note that, using (16),  $I'_k(t)$  is expressed as follows

$$I'_k(t) = \sum_{i=0}^{I-1} (I_{k,i} - I_{k,i-1}) \delta(t - t_i). \quad (36)$$

The startup and shutdown cost term of generating unit  $k$  over the day-ahead horizon  $\mathcal{T}$  is derived by integrating  $C_k^I(I'_k(t); t)$  and substituting the expressions for  $I'_k(t)$ :

$$\begin{aligned} \int_{\mathcal{T}} C_k^I(I'_k(t); t) &= \int_{\mathcal{T}} \sum_{i=0}^{I-1} \delta(t - t_i) \max \left( c_k^{(\text{SU})} (I_{k,i} - I_{k,i-1}), \right. \\ &\quad \left. - c_k^{(\text{SD})} (I_{k,i-1} - I_{k,i}) \right) dt = \\ &= \sum_{i=0}^{I-1} \max \left( c_k^{(\text{SU})} (I_{k,i} - I_{k,i-1}), -c_k^{(\text{SD})} (I_{k,i-1} - I_{k,i}) \right) \end{aligned} \quad (37)$$

The  $\max(\cdot)$  term in (37) is nonlinear, so we substitute it with the positive variable  $C_{k,i}^\delta$  that is linearly constrained as:

$$C_{k,i}^\delta \geq c_k^{(\text{SU})} (I_{k,i} - I_{k,i-1}), \quad (38)$$

$$C_{k,i}^\delta \geq c_k^{(\text{SD})} (I_{k,i-1} - I_{k,i}), \quad \forall k, \forall i. \quad (39)$$

### B. Joint Generation and Ramping Cost Function

The introduction of continuous-time ramping trajectory in (26) provides a natural way to monetize the ramping capacity provided by the generating units in the day-ahead market. Here, we define the instantaneous joint energy generation and ramping cost function of generating units,  $C_k^{G,G'}(G_k(t), G'_k(t))$ , which is a function of generation trajectory and ramping trajectory variables of units, and thus a surface in the three-dimensional space. The cost also depends to change of commitment variable. We assume that the joint cost function is increasing with respect to both of the variables. Extending the current quadratic generation cost function of generating units, a choice for the instantaneous joint energy generation and ramping cost function of generating unit  $k$  is a elliptic paraboloid of form:

$$C_k^{G,G'}(G_k(t), G'_k(t)) = a(G_k(t) + b)^2 + c(G'_k(t) + d)^2 - ac\rho(G_k(t) + b)(G'_k(t) + d) \quad (40)$$

defined for  $\underline{G}_k \leq G_k(t) \leq \bar{G}_k$  and  $\underline{G}'_k \leq G'_k(t) \leq \bar{G}'_k$ , and  $a, b, c, d, \rho$  are constant coefficients. The task here is to linearize the quadratic surface cost function so the UC problem would remain linear with respect to the variables. Various methods are developed in [23] for obtaining a piecewise linear approximation of general functions of two variables, all of which introduce additional binary variables in the model. We leverage the continuity and convexity properties of elliptic paraboloid and linearize the cost function without introducing additional binary variables.

As shown in Fig. 1, we start linearization of joint cost function by tiling the plain created by  $G_k(t)$  and  $G'_k(t)$ . For this, let us divide the generation capacity of generating unit  $k$ , i.e. the vertical coordinate in Fig. 1, to  $N_k$  sections using intermediate generation points  $g_0 = \underline{G}_k, g_1, \dots, g_{N_k} = \bar{G}_k$ . Let us also divide the up (positive) ramping capacity of generating unit  $k$  to  $J_k^+$  sections using intermediate ramping points  $g_0^+ = 0, g_1^+, \dots, g_{J_k^+}^+ = \bar{G}'_k$ , and the down ramping capacity to  $J_k^-$  sections using points  $g_0^- = 0, g_1^-, \dots, g_{J_k^-}^- = \underline{G}'_k$ . This would divide the up ramping quarter-plane to  $N_k J_k^+$  tiles, and the down ramping quarter-plane to  $N_k J_k^-$  tiles.

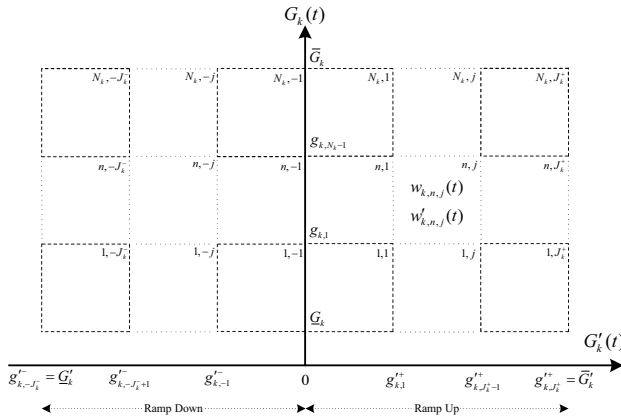


Fig. 1. Generation-ramping plane tessellation

Defining positive continuous-time auxiliary generation variables  $w_{k,n,j}(t)$  to model the generation at each tile of the half plane, the generation trajectory of unit  $k$  can be written as:

$$G_k(t) = \underline{G}_k I_k(t) + \sum_{n=1}^{N_k} \sum_{j=-J_k^-}^{J_k^+} w_{k,n,j}(t). \quad (41)$$

Similarly, defining the positive continuous-time auxiliary ramping variables  $w'_{k,n,j}(t)$  for  $j = -J_k^-, \dots, J_k^+$ , the ramping trajectory  $G'_k(t)$  can be written as follows:

$$G'_k(t) = \sum_{n=1}^{N_k} \sum_{j=1}^{J_k^+} w'_{k,n,j}(t) - \sum_{n=1}^{N_k} \sum_{j=-J_k^-}^{-1} w'_{k,n,j}(t). \quad (42)$$

The auxiliary variables  $w_{k,n,j}(t), w'_{k,n,j}(t)$  are constrained in the generation-ramping plain by:

$$0 \leq w_{k,n,j}(t) \leq g_{k,n+1} - g_{k,n}, \quad \forall k, \forall n, \forall j \quad (43)$$

$$0 \leq w'_{k,n,j}(t) \leq g_{k,j+1}^- - g_{k,j}^-, \quad \forall k, \forall n, \forall j. \quad (44)$$

Using the change of variables to auxiliary variables in (41) and (42), the joint cost function (40) is linearized in terms of the auxiliary variables as follows:

$$\begin{aligned} C_k^{G,G'}(G_k(t), G'_k(t), I_k(t)) &= C_k^{W,W'}(W_k(t), W'_k(t), I_k(t)) \\ &= C_k(\underline{G}_k, 0) I_k(t) + \sum_{n=1}^{N_k} \sum_{j=-J_k^-}^{J_k^+} (c_{k,n,j}(t) w_{k,n,j}(t) + c'_{k,n,j}(t) w'_{k,n,j}(t)) \end{aligned} \quad (45)$$

where  $W_k(t) = (w_{k,n,j}(t))$  and  $W'_k(t) = (w'_{k,n,j}(t))$  are the vectors of auxiliary generation and ramping variables for all  $n, j$ ;  $C_k(\underline{G}_k, 0)$  is the cost of generating minimum power at zero ramping, and  $c_{k,n,j}(t)$  and  $c'_{k,n,j}(t)$  are the energy and ramping cost coefficients defined for the tiles in the generation-ramping half plane of unit  $k$ . In order to express the cost function in the Bernstein space, let us expand the generation and ramping auxiliary variables in the space spanned respectively by the Bernstein polynomials of degree 3 and degree 2 as follows:

$$w_{k,n,j}(t) = (\mathbf{w}_{k,n,j})^T \mathbf{e}^{(3)}(t) = \sum_{p=0}^{P-1} w_{k,n,j,p} e_p^{(3)}(t) \quad (46)$$

$$w'_{k,n,j}(t) = (\mathbf{w}'_{k,n,j})^T \mathbf{e}^{(3)}(t) = \sum_{p=0}^{P-1} w'_{k,n,j,p} e_p^{(3)}(t) \quad (47)$$

where  $\mathbf{w}_{k,n,j}$  and  $\mathbf{w}'_{k,n,j}$  represent the vectors of the Bernstein coefficients in interval  $i$ . Using (41) and (42), the coefficients of the total generation and ramping trajectories are:

$$\mathbf{G}_k = \underline{G}_k \mathbf{I}_k + \sum_{n=1}^{N_k} \sum_{j=-J_k^-}^{J_k^+} \mathbf{w}_{k,n,j}, \quad (48)$$

$$\mathbf{G}'_k = \sum_{n=1}^{N_k} \sum_{j=1}^{J_k^+} \mathbf{w}'_{k,n,j} - \sum_{n=1}^{N_k} \sum_{j=-J_k^-}^{-1} \mathbf{w}'_{k,n,j}. \quad (49)$$

Substituting the Bernstein expansions of the auxiliary variables and commitment variable in the cost function (45), and integrating over the scheduling horizon  $\mathcal{T}$ , we have:

$$\int_{\mathcal{T}} C_k^{W,W'}(W_k(t), W'_k(t), I_k(t)) dt = \sum_{i=0}^{I-1} \left[ C_k(\underline{G}_k, 0) \sum_{q=0}^3 I_{k,n,4i+q} \int_{\mathcal{T}_i} e_{4i+q}^{(3)}(t) dt + \sum_{n=1}^{N_k} c_{k,n,j,i} \sum_{q=0}^3 w_{k,n,j,4i+q} \int_{\mathcal{T}_i} e_{4i+q}^{(3)}(t) dt + \sum_{n=1}^{N_k} c'_{k,n,j,i} \sum_{q=0}^2 w'_{k,n,j,3i+q} \int_{\mathcal{T}_i} e_{3i+q}^{(2)}(t) dt \right] \quad (50)$$

where the cost coefficients  $c_{k,n,j,i} \equiv c_{k,n,j}(t_i)$  and  $c'_{k,n,j,i} \equiv c'_{k,n,j}(t_i)$  are constant in each interval  $i$ , and the integrals of Bernstein basis functions of degree 2 and 3 are calculated as:

$$\int_{\mathcal{T}_i} e_{3i+q}^{(2)}(t) dt = T_i \int_0^1 b_{q,2}(t) dt = \frac{T_i}{3}, \quad q = 0, 1, 2 \quad (51)$$

$$\int_{\mathcal{T}_i} e_{4i+q}^{(3)}(t) dt = T_i \int_0^1 b_{q,3}(t) dt = \frac{T_i}{4}, \quad q = 0, 1, 2, 3. \quad (52)$$

Using the integrals (51), (52) and substituting  $I_{k,n,4i+q}$  from (35), the joint energy generation and ramping cost function of generating unit  $k$  over the day-ahead scheduling horizon,  $C_{k,\mathcal{T}}^{G,G'}(\cdot)$ , is expressed as follows:

$$C_{k,\mathcal{T}}^{W,W'}(\mathbf{W}_k, \mathbf{W}'_k, \mathbf{I}_k) = \sum_{i=1}^{I-2} T_i C_k(\underline{G}_k, 0) I_{k,i} + C_k(\underline{G}_k, 0) \left( \frac{T_0 I_{k,0}}{2} + \frac{T_{I-1} I_{k,I-1}}{2} \right) + \sum_{i=0}^{I-1} T_i \sum_{n=1}^{N_k} \frac{c_{k,n,j,i}}{4} \sum_{q=0}^3 w_{k,n,j,4i+q} + \sum_{i=0}^{I-1} T_i \sum_{n=1}^{N_k} \frac{c'_{k,n,j,i}}{3} \sum_{q=0}^2 w'_{k,n,j,3i+q}, \quad (53)$$

where  $\mathbf{W}_k = (\mathbf{w}_{k,n,j})$  and  $\mathbf{W}'_k = (\mathbf{w}'_{k,n,j})$  are respective vectors of the coefficients of the auxiliary generation and ramping variables for all  $n, j$ . If we choose to set the ramping cost coefficients to zero, the cost function in (53) becomes the classic cost function of units which is a function of only generation trajectory.

## V. THE PROPOSED UNIT COMMITMENT MODEL

In this section, we propose the UC formulation which represents the discrete-time approximation of the variational problem (5)-(8), using the representation of continuous-time generation and ramping trajectories respectively in the function spaces of Bernstein polynomials of degree 3 and 2 in (24) and (26), over  $I$  intervals in the scheduling horizon  $\mathcal{T}$ . In particular, the continuous-time generation trajectories  $G_k(t)$  is represented by the coefficients vector  $\mathbf{G}_k$ . The continuous-time ramping trajectory  $G'_k(t)$  is represented through (27) in terms of the coefficients of generation trajectory as  $\mathbf{G}'_k = \mathbf{O}\mathbf{G}_k$ . The continuous-time binary commitment variable is

also represented using the coefficients  $\mathbf{I}_k$  in (34). In the following, we present the proposed discrete-time UC model which is formulated as a mixed-integer linear programming (MILP) problem on variables  $\mathbf{G}_k$  and  $\mathbf{I}_k$ .

1) *Objective Function*: As presented in the original variational problem (5), the objective of the UC problem is to minimize the total continuous-time operation cost of generating units over the scheduling horizon, which is derived in terms of the coefficients of commitment variables as well as auxiliary generation and ramping variables in (53). Thus, the objective function is formulated as the summation of the joint generation and ramping cost function in (53) over all generating units

$$\min \sum_K C_{k,\mathcal{T}}^G(\mathbf{G}_k, \mathbf{G}'_k, \mathbf{I}_k). \quad (54)$$

The objective function is subject to (38) and (39) which govern the startup/shutdown cost in the objective function. The relationship between the coefficients of generation and ramping trajectories with the associated auxiliary variables is respectively presented in (48) and (49), while the relation between the coefficients of generation and ramping trajectories is presented in (27). The convex hull property of Bernstein polynomials, explained in Section III.C, allows us to express the continuous-time bounds on auxiliary variables in (43)-(44), in terms of finite number of constraints on the coefficients as:

$$0 \leq w_{k,n,j,4i+q} \leq g_{k,n+1} - g_{k,n}, \quad \forall k, n, q, j \quad (55)$$

$$0 \leq w'_{k,n,j,3i+q} \leq g'_{k,j+1} - g'_{k,j}, \quad \forall k, n, q, j. \quad (56)$$

2) *Balance and Generation Continuity Constraints*: The discrete-time equivalent of the continuous-time load generation balance constraint in (6) is represented in (57), where the Bernstein coefficients of generation trajectory of the generating units sum up to balance the corresponding load coefficients. In essence, unlike the current UC models where the units are scheduled to balance the hourly samples of load, (57) would schedule the continuous-time generation trajectory to balance the continuous-time variations of net-load within the intervals, as represented by the Bernstein coefficients. In addition, the constraints (28) ensure the  $C^1$  continuity of the generation trajectory over the scheduling horizon.

$$\sum_{k=1}^K \mathbf{G}_k = \mathbf{N} \quad (57)$$

3) *Generation Capacity and Ramping Constraints*: As mentioned in Section III.B, the convex hull property of Bernstein polynomials allow us to enforce the generation capacity constraint in continuous-time by capping the Bernstein coefficients of the generation trajectory as follows:

$$\mathbf{G}_k \geq \underline{G}_k \mathbf{I}_k \quad (58)$$

$$\mathbf{G}_k \leq \overline{G}_k \mathbf{I}_k. \quad (59)$$

The continuous-time ramping constraints can be applied in a similar way by capping the Bernstein coefficients of the continuous-time ramping trajectory of generating units in (26), only two of which are independent in each interval due to the  $C^0$  continuity of ramping trajectory. The ramping up and down constraints for the first Bernstein coefficient of generation

ramping trajectory at each interval,  $G'_{k,3i}$ , accounting for the startup and shutdown ramp limits, are

$$G'_{k,3i} \leq \bar{G}'_k I_{k,(i-1)} + G'^{(SU)}_k (I_{k,i} - I_{k,(i-1)}) + \bar{G}_k (1 - I_{k,i}) \quad \forall k, \forall i \quad (60)$$

$$-G'_{k,3i} \leq \underline{G}'_k I_{k,i} + G'^{(SD)}_k (I_{k,(i-1)} - I_{k,i}) + \bar{G}_k (1 - I_{k,(i-1)}) \quad \forall k, \forall i, \quad (61)$$

where  $G'^{(SU)}_k$  and  $G'^{(SD)}_k$  respectively represent the startup and shutdown ramp limits of generating unit  $k$ . The ramping up and down constraints for the second Bernstein coefficient of generation ramping trajectory,  $G'_{k,3i+1}$ , are

$$G'_{k,3i+1} \leq \bar{G}'_k I_{k,i} \quad \forall k, \forall i = 0 \dots I-2 \quad (62)$$

$$-G'_{k,3i+1} \leq \underline{G}'_k I_{k,i} + \eta(1 - I_{k,i+1}) \quad \forall k, \forall i = 0 \dots I-2 \quad (63)$$

where  $\eta$  is a sufficiently large constant and assures that the constraint does not prevent the unit from turning off.

4) *Minimum on/off Time Constraints*: the minimum on and minimum off time constraints of generating units are formulated as follows:

$$\sum_{i'=i}^{i+T_k^{(on)}-1} T_{i'} I_{k,i'} \geq T_k^{(on)} (I_{k,i} - I_{k,(i-1)}) \quad (64)$$

$$\sum_{i'=i}^{i+T_k^{(off)}-1} T_{i'} (1 - I_{k,i'}) \geq T_k^{(off)} (I_{k,(i-1)} - I_{k,i}) \quad (65)$$

where  $T_k^{(on)}$  and  $T_k^{(off)}$  represent the minimum on and off times of generating unit  $k$ .

## VI. NUMERICAL RESULTS

To analyze and compare the UC formulations we use the data regarding 32 generating units included in the IEEE Reliability Test System (RTS) [24] and load data from the CAISO. In Cases 1 and 2, we respectively study and analyze the results of running the current day-ahead (DA) UC model and our proposed UC with continuous-time generation and ramping trajectory models on the IEEE-RTS and CAISO load data. In both cases, we also simulated the real-time (RT) economic dispatch in five-minute intervals, which schedules for the deviations of day-ahead dispatch from the real-time five-minute load forecast data. We took the five-minute net-load forecast data of CAISO for Feb. 2, 2015, scaled it down to the original IEEE-RTS peak load of 2850MW, and generated the hourly day-ahead load forecast where the forecast standard deviation is considered to be %1 of the load at the time. The two DA load profiles and their deviation from the RT load are shown in Fig. 2.a) and Fig. 2.b) respectively. The impact of solar generation on reducing the CAISO's load during sunlight and the resulting ramping events is obvious in Fig. 2.a).

The DA and RT simulation results for both cases are summarized in Table I. In Table I, the DA operation cost in the proposed UC model is increased by \$5,095.7, while the RT operation cost is reduced by \$10,651.6 (%63) as compared to Case 1, resulting in the total reduction of \$5,555.9 in daily operation cost in Case 2. In Fig. 2.b), the piecewise constant load profile used in traditional UC model leaves out

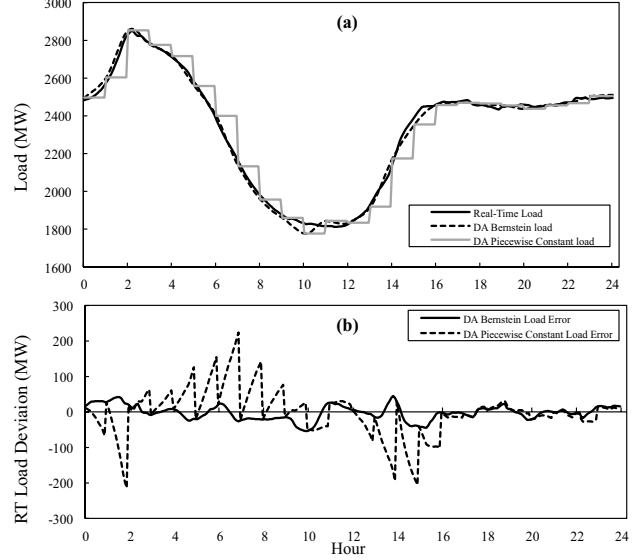


Fig. 2. a) DA load profiles in Cases 1 and 2, b) RT deviation from DA load profiles.

a substantial amount of net-load for RT operation. The net-load presents several fast ramping events specially when the solar generation starts to rise in the early morning and suddenly drops during sunset. The substantial load deviation and several fast ramping events causes the relatively high RT operation cost for Case 1 in Table I. In addition, due to the lack of ramping capacity in RT operation, 27 ramping scarcity events are observed in Case 1; that is, the RT economic dispatch becomes infeasible due to insufficient ramping capacity of generation units, which reveals the inadequacy of the current UC model in accounting the sub-hourly variations of net-load. However, no violations of the power balance is observed in the RT operation of Case 2, which demonstrates the ability of the proposed UC model to effectively schedule the ramping capability of units to cater to the fast ramping of the net-load.

TABLE I  
SCHEDULING RESULTS

Case	DA Operation Cost (\$)	RT Operation Cost (\$)	Total DA and RT Operation Cost (\$)	RT Ramping Scarcity Events
Case 1	471,130.7	16,882.9	488,013.6	27
Case 2	476,226.4	6,231.3	482,457.7	0

The continuous-time generation trajectories for two cases are shown in Figs. 3, where the units are grouped to 9 groups with various capacities, costs and characteristics. In Fig. 3.a), the current hourly UC model provides a constant hourly schedule for the generating units and results in a piecewise constant generation trajectory. In Fig. 3.b), the proposed UC model provides a continuous-time schedule for generating units which efficiently utilizes their ramping capability to follow the continuous-time variations of the net-load, while leaving less energy to schedule in the RT operation. In Case 1, a total of twenty units are committed, while in the proposed model in Case 2, additional five units are committed to secure adequate ramping capacity in the hours 1-3 when there is a fast ramp in the net-load. Moreover, in Case 2, the 197MW units

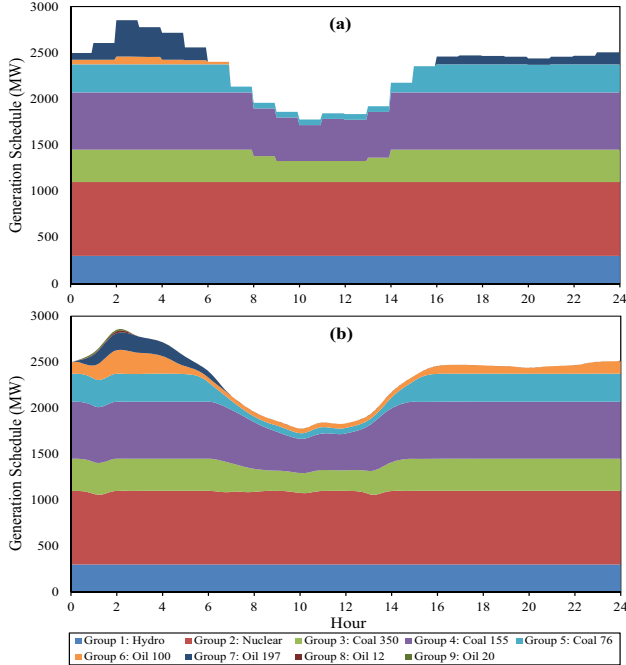


Fig. 3. DA generation trajectory in a) Case 1, b) Case 2.

are not committed in the hours 16-24; instead the 100MW units with more than twice the ramping capacity are kept on to also supply the fast ramping of net-load caused by solar generation during hours 6-16. This result highlights that ramping scarcity events are observed partly due to the fact that the set of online generation fleet does not provide enough ramping capacity in order to supply the ramping requirements of the net-load in real-time

The continuous-time ramping requirement of net-load and its breakdown to the scheduled ramping trajectory of generating units are shown in Fig. 4. In Fig. 4, the proposed UC model accounts for continuous-time ramping of load, which manifest several sub-hourly spikes, and schedules the generating units in day-ahead to deliver the continuous-time ramping requirement of load in real-time operation.

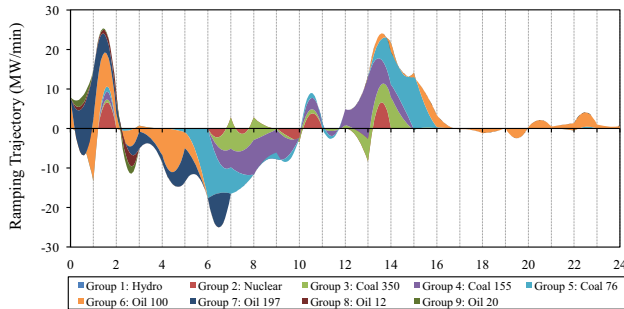


Fig. 4. Continuous-time ramping trajectory of units in Case 2

In order to evaluate the performance of our proposed model in different loading and forecast error conditions, we repeated the same Cases 1 and 2 in Figs. 5 for the CAISO's load data of the entire month of Feb. 2015, and also added the results obtained from the traditional UC model with 48 half-hour

periods. From the scatter diagram in Fig. 5.a), we can clearly see that our proposed UC model outperforms the other two cases in terms of real-time and total operation cost reduction, even compared to the half-hourly UC solution. Note that the latter could have benefited from having twice the binary variables for half-hourly commitment status changes, which is prevented in this test for our model; this indicates that the brute-force solution of increasing the number of scheduling intervals is inferior compared to our solution. In addition, Fig. 5.b) reveals that our proposed UC model results in much fewer ramping scarcity events.

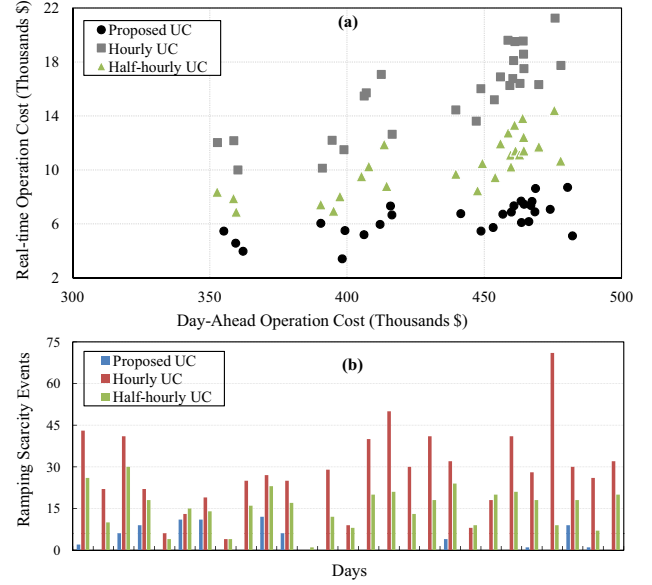


Fig. 5. Simulation of CAISO's load of Feb. 2015, a) DA vs RT operation costs, b) number of RT ramping scarcity events

**Ramping Cost Consideration:** With the joint energy generation and ramping cost function, assuming that the up/down ramping cost coefficients equal to 10% of the highest incremental cost of generating units, we solved the proposed UC model without and with the joint cost function, for the CAISO's load data of Feb. 3, 2015. The DA operation cost with the joint cost function rises up to \$473,469.2, as compared to \$470,374.9 when we considered the classic generation cost function. In order to compare the two cost, we calculated the ramping cost afterwards using the same ramp cost coefficients, which equals to \$3,639.3, resulting in total cost of \$474,014.1 in the case in which we did not integrate the joint cost in the UC problem. Hence, integrating joint generation and ramping cost function in the UC problem further reduces the system cost.

#### A. Computation Time

The computation time of simulating the day-ahead operation of IEEE-RTS with 32 generating units for the CAISO load data of Feb. 2, 2015, using the traditional 24-hour UC model, 48 half-hourly UC model, and the proposed function space-based UC model are respectively 0.257s, 0.572s, and 1.369s, while the upper bound on the duality gap is set to be zero.

The study cases were solved using CPLEX 12.2 [25] on a desktop computer with a 2.9GHz i7 processor and 16 GB of RAM. Our proposed UC model has the same number of binary variables as compared to the traditional 24-hour UC model, but the reason for increased computation time is that it includes additional continuous variables, and equality and inequality constraints. The number of continuous generation variables is increased from 1 to 4 in each interval for each generating unit. The number of equality balance constraints is increased from 1 to 4 equality constraints in each interval. The number of inequality capacity and ramping constraints is increased from 2 and 2, respectively to 8 and 6 constraints in each interval for each generating unit. There are also additional two constraints in each interval for each generating unit enforcing the  $C^1$  continuity of the generation trajectory. However, having the same number of binary variables is promising for large-scale implementation of our proposed model. In fact, the computation time of a MILP problem, due to the nature of branch-and-cut algorithm, is almost an exponential function with respect to the number of integer variables [26].

## VII. CONCLUSION

We propose to approximate the continuous-time formulation of the UC problem using a function space model, expanding power trajectories on the basis of Bernstein polynomials of degree 3 and suggest a new form for the market bids that describe the joint generation and ramping cost, to promote offers of greater flexibility in ramping from generating units. The proposed model preserves the MILP structure of current UC practice, with the same number of binary variables in the problem. Our numerical results on CAISO's real load data show that the commitment and schedule of the units in the proposed model is different from those of the current practice and that the application of proposed UC model has the potential of reducing significantly the number of ramping scarcity events in the real-time operation and of reducing the total operation costs. Not considered in this work is how to appropriately tune the net-load forecasts and deal with uncertainty in the proposed UC model, which are natural extensions of the current framework.

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